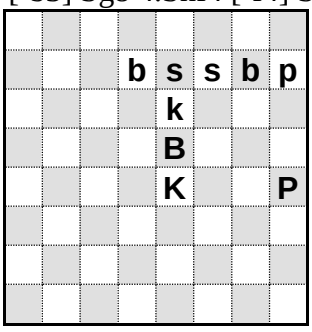
Notes on the Frolkin-Tylor tiger task entry – Chris Tylor, 10-10-19.
Our game, as given below, starts from the postion with wKe4 and contains four model and ideal mates (marked yellow).
1.Bd1 Bd7 2.Sd5+ Ke6 3.Shf4[-f4] Sg8 4.Bg4[-g4] Se7 5.Sf4[-f4] Sd6+ 6.Kf4 Kf7 7.Kg5 Sg6 8.h5 Se4[-e4] 9.Bd4 h6[-h6] 10.h6 Bg4 11.h7 Bf6+ 12.Kh6 Se7 13.Be3 Sf5[-f5] 14.Bg5 Bg7[-g7]15.h8Q Bf3 16.Kh7 Bh5 17.Bh6 Bg6[-g6] 18.Qd4 Ke6 19.Qd5+ Kf6 20.Bg5[-g5] Ke7 21.Kg6 Kf8 22.Qd8[-d8]

We used several different approaches before discovering the family of games that includes this one. First, we considered the ending, identifying the general points that the final mate must be delivered by a promoted P, that B and Q of the same colour appeared the most likely combination of pieces to deliver the two final mates, and that a P on the h-file would give the best opportunities for mates by the other side while it was advancing to promotion. (We went further, discovering a number of interesting sequences of quick mates leading up to a good ending, but were unable to work any of these into short complete games.) Next, realising that the critical part of any game must be the P mate that allows the opposing P to advance to promotion, we identified an opening sequence with such a P mate on move B4, but were unable to produce any short games from this sequence.

After this largely unsuccessful work,we turned to the more prosaic approach of repeatedly using Francois Labelle's solving program *Jacobi* with the condition '#RChess' to identify sequences of short mates from successive starting positions, and quickly found that four such sequences led us almost inevitably to a 21.5-move game – suggesting that anyone else using *Jacobi* would almost certainly duplicate our main results!

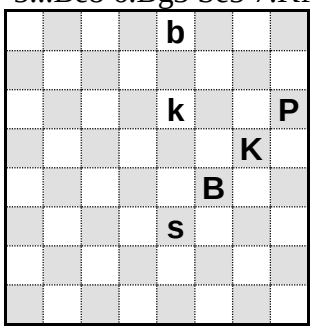
Our four series of *Jacobi* tests can be summarised as follows.
1. From most of the possible starting positions, testing for the shortest possible 3-mate sequences leads quickly to the command 'stip h$2.5 stip h$1 stip h$1', which gives just 32 sequences with only 2 final positions. One of these appears to give good possibilities for a quick bP mate leading to a wP promotion – suggesting that the preferred starting position would have wKe4, and leading to position A below. A somewhat different set of moves from that in our entry game is 1.Bd1 Bd7 2.Sg6+ Ke6 3.Bb3 [-b3] Sg8 4.Shf4 [-f4] Se7 5.Sf4 [-f4].



Position A, after W5.

2. Finding a suitable sequence of mates from position A is the only stage of the procedure that is in any way difficult, partly because the more promising Jacobi commands give large numbers of solutions which must be scanned visually to identify any with an advanced passed wP. (Such scanning might be impracticable without *Jacobi*'s facility for displaying the final position of a sequence of mates.) Commands starting with 'stip h$2.5' lead to bP mates and wP advances which can result in 22.5-move games, but delaying the opening mate gives even better results. Thus the command 'stip h$3.5 stip h$2

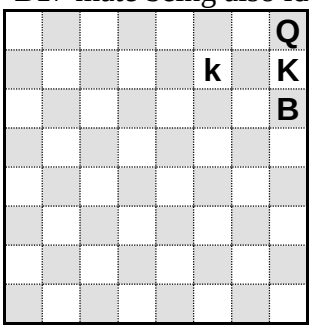
stip h$1’ gives an apparently daunting 775 solutions, but scanning their final positions soon reveals though nearly all are unsuitable, just 3 lead to position B with the wP advanced to h6. One sequence is 5...Be8 6.Bg3 Se5 7.Kf4 Sd5+ 8.Kg5 h6 [-h6] 9.Bf4 Se3 10.h5 Sf3 [-f3] 11.h6 Bf6 [-f6].



Position B, after B11

Further tests involving the scanning of even more solutions show that this position (with bSe3 on either e3, e5 or f6) can also be reached from position A by sequences with mates 4 and 5 on B8 and B9 or on B9 and B10. (But note that our entry game does not go through position B at all.)

3. Finding a suitable 2-mate sequence from position B turns out to be unexpectedly easy. The command ‘stip h$3 stip h$2’ gives 211 solutions with no further P advances, but ‘stip h$3 stip h$3’ gives 411 solutions in all of which the wP has promoted, and from which position C (which we had not even considered in our earlier work on endings!) is the obvious target. One sequence is 12.h7 Sc4 13.Kh6 Sd6 14.Bg5 Sf5 [-f5] 15.h8=Q Bh5 16.Kh7 Kf7 17.Bh6 Bg6 [-g6], in which both mates are models, the B17 mate being also ideal.



Position C, after B17

4. Concluding the game from position C is easy. *Jacobi* tests on the shortest possible sequences from position C quickly lead to ‘stip h$2.5 stip h$2’ with 12 solutions, all valid, and half with pairs of ideal mates (as in our entry game).

After discovering our first 21.5-move game we carried out further tests starting from position A (and related positions), many of which tests resulted in very large numbers of solutions to be scanned. We failed to find any shorter games or any improvement on 4 model mates, but did discover two alternative 21.5-move game sequences. One of these reaches position C while avoiding position B; it allows the final four mates to be ideal as well as model, and we therefore used it in our game entry (see above). The other sequence follows this one up to move B9, but avoids both positions B and C; it has Q promotion on move W13, but a slower finish and an impure mate 7. It can run 5...Sd6+ 6.Kf4 Kf7 7.Kg5 Sg6 8.h5 Se4 [-e4] 9.Bg3 h6 [-h6] 10.h6 Sf4 11.Kh4 Bf6 [-f6] 12.h7 Kg6 13.h8=Q Sg2 [-g2] 14.Qh6+ Kf5 15.Kh5 Bc6 16.Bh4 Bf3 [-f3] 17.Qc1 Ke6 18.Kg5 Kd7 19.Qc6+ Kd8 20.Kg6 [-h4] Ke7 21.Qd5 Kf8 22.Qd8 [-d8].